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Study on the Sliding Mode Fault Tolerant Predictive Control Based on Multi Agent Particle Swarm Optimization

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Abstract: For a class of uncertain discrete-time systems with time varying delay, the problem of robust fault-tolerant control for such systems is studied by combining the design of sliding mode control (SMC) and model predictive control (MPC). A sliding mode fault tolerant predictive control based on multi agent particle swarm optimization (PSO) is presented, and the design, analysis and proof of the scheme are given in detail. Firstly, the sliding mode prediction model of the system is designed by assigning poles of the output error of the system. The model has time varying characteristics, and it can improve the motion quality of the system while ensuring the sliding mode is stable. Secondly, a new discrete reference trajectory considering time-delay systems subjected simultaneously to parameter perturbations and disturbances is proposed, which not only can ensure that the state of the system has good robustness and fast convergence in the process of approaching sliding mode surface, but also can inhibit chattering phenomenon. Thirdly, the multi agent PSO improves the receding-horizon optimization, which can quickly and accurately solve the control laws satisfying the input constraints, and can effectively avoid falling into local extrema problem of the traditional PSO. Finally, the theoretical proof of robust stability of the proposed control scheme is given. Experimental results of quad-rotor helicopter semi physical simulation platform show that the state of uncertain discrete-time systems with time varying delay is stable under the action of the proposed control scheme in this paper. The advantages of fast response, less overshoot and small control chattering prove the feasibility and effectiveness of the proposed control scheme.

Keywords: PSO, quad-rotor helicopter, sliding mode fault tolerant predictive control, time varying delay, uncertain discrete-time systems.

1. INTRODUCTION

With the rapid development of computer technology and the actual needs of industrial automation and other fields, the analysis and design of discrete control system have become an important part of control theory. In engineering practice, there are some errors in the process of modeling, and the physical structure of the system will be influenced by the working conditions. At the same time, there is inevitable external interference. All these uncertainties will have a profound impact on the final control effect of the discrete control system. In addition, with the increasing complexity of the structure of the actual discrete control system, the large time delay is introduced in the process of signal transmission, computation and remote control. The existence of time delay will make the analysis and design of systems become more complex and difficult. Especially for aerospace, precision machining and other fields with fast response and high accuracy requirements, the control accuracy of the system will be greatly reduced, which may even cause the system instability. With the variety of tasks of the control system and the complexity of the structure, faults of sensors, actuators, and the internal components of the system are inevitable when the system is running. Therefore, it has become an urgent problem for engineering application to investigate and analyze the fault tolerant control algorithm suitable for uncertain discrete-time systems with time varying delay [1].

SMC has robustness to the uncertainties of parameter perturbation and external disturbance, so it is widely used in the control of uncertain discrete systems [2–4]. However, when there is a time delay in the discrete system, SMC shows obvious performance degradation. Especially when the time delay is too large, and the system has a high demand for fast response, SMC is often difficult to meet

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the requirements of the actual control. Compared with SMC, MPC can use its own prediction and optimization ability to estimate the system performance of a period of time in the future, and then get a real-time control strategy of online optimization. MPC is more suitable for eliminating the influence of time delay on the performance of the discrete system [5, 6]. Therefore, for the uncertain discrete-time systems with time delay, combining with SMC and MPC not only can make full use of advantages of good robustness of SMC for dealing with uncertainties, but also can optimize the performance of the control to avoid the effect of time delay on the stability of the system.

At present, although the sliding mode predictive control (SMPC) has become a feasible method to solve the problem of robust control for uncertain discrete-time systems [7–9], researches and applications are still lack of in-depth for the problem of the existence of time delay. The control requirements of robust control of uncertain mismatched systems can be satisfied under SMPC, but this scheme is only applied to the single input single output system. A SMPC control scheme is proposed for robust control of multi input multi output (MIMO) systems in [10]. However, this method considers the modeling error only and can not overcome the adverse effect caused by the external interference in the actual engineering system. A SMPC based on Lazy Learning is designed for MIMO systems in [11], which has strong adaptability and capacity of resisting disturbance. However, the hierarchical search strategy used in this scheme often introduces large time delay, and then reduces the control performance in practical application. A fast and accurate positioning SMPC scheme that can effectively reduce the position tracking error is proposed for Micro/Nano-position servo systems in paper [12]. But the performance of this scheme is mainly depended on the performance of the observer and it is unable to maintain good control performance when the parameters of the system change. The design of the observer is optimized in [13] based on [12], and the chattering phenomenon is suppressed by introducing integral term in the design process of the control law. But it is still unable to avoid the effect of observer on the control performance. A SMPC control method without the state observer is designed in [14], but determination process of the model parameters is complex and the selection of some parameters relies on experiences. SMPC is used in the attitude control of hypersonic vehicle in [15], which can ensure that the system has robustness to the uncertainties. However, this method has obvious chattering phenomenon in the simulation experiment, which is not conducive to engineering application. The chattering phenomenon is weakened by improving the reference trajectory in [16]. Although the above literatures are better to solve the uncertainties in the discrete-time system, the effect of time delay on the system is not considered. The time delay of the network control system is well dealt with in [17]. But SMC and MPC are independently designed, which can not guarantee the mutual influence between the two kinds of control. A SMPC scheme with time delay compensator is designed for aero-engine control system in [18], which can compensate the effect of delay time well. But the time delay considered is fixed and time varying delay is more useful in practical systems. For a class of uncertain discrete systems with external disturbances, a new method based on global sliding mode surface prediction model is proposed in paper [20], in which the design of reference trajectory based on power reaching law effectively inhibites the chattering problem of sliding mode and the system under the control scheme has robustness. However, that control scheme does not consider faults and the system time delay, so it is unsatisfactory when it is applied to a practical system.

In this paper, the problem of fault tolerant control for uncertain discrete systems with time varying delay is studied and a new scheme of fault tolerant control based on multi agent PSO is proposed. The multi agent PSO is designed to ensure the receding-horizon optimization fast and accurate. A new type of reference trajectory is proposed which can effectively suppress the chattering phenomenon. The reachability of the discrete sliding mode control law is proved. Finally, the effectiveness of the proposed control scheme is verified by the quad-rotor helicopter semi physical simulation.

The rest of this paper is organized as follows: The object model is proposed in Section 2. Controller design, stability analysis and algorithm implementation steps are discussed in detail in Section 3. Finally, the simulation results of the quad-rotor helicopter semi-physical simulation platform are presented in Section 4 and the conclusion of this paper is in Section 5.

2. FORMULATION AND PRELIMINARIES

The state-space model of the system with internal perturbations, external disturbances and the time varying delay under actuator faults is considered as following equation:

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)u(k) \\ \qquad\qquad + (A_d+\Delta A_d)x(k-\tau(k)) \\ \qquad\qquad + v(k) + Ef(k), \\ y(k) = Cx(k), \end{cases} \tag{1}$$

where $x(k) \in R^n$ is the state of the system, $u(k) \in R^p$ is the input of the system, $y(k) \in R^q$ is the output of the system, $A \in R^{n\times n}$, $B \in R^{n\times p}$, $A_d \in R^{n\times n}$ and $E \in R^{n\times m}$ are constant matrices, ΔA, ΔB and ΔA_d are the internal perturbation of the system, $\tau(k) \in R^n$ is external disturbance of the system, $\tau(k) \in R^+$ is the time varying delay of the system and $f(k) \in R^m$ is fault function. System (1) can be rewritten as following:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + A_d x(k-\tau(k)) + d(k), \\ y(k) = Cx(k), \end{cases} \tag{2}$$

where

$$\begin{aligned} d(k) =& \Delta Ax(k) + \Delta Bu(k) + \Delta A_d x(k-\tau(k)) \\ &+ v(k) + Ef(k). \end{aligned} \tag{3}$$

Assumption 1: The rate of change of the fault and uncertainty of the system is bounded.

$$|d(k) - d(k-1)| \le d_0.$$

Assumption 2: The fault and uncertainty of the system have an upper and lower bound as follow:

$$d_L \le |d(k) \le d_U|.$$

3. CONTROLLER DESIGN AND STABILITY ANALYSIS

3.1. Sliding mode fault-tolerant predictive controller design

3.1.1 Prediction model design

Define system output error as (4).

$$e(k) = y(k) - y_r(k), \tag{4}$$

where $y_r(k)$ is the desired output, $y(k)$ is actual output.

The linear sliding surface $s(k) = \sigma e(k)$ is adopted, and $\sigma = [\sigma_1, \sigma_2, \cdots, \sigma_q]$ is designed by pole assignment. The sliding mode prediction model based on the output error is (5).

$$s(k+1) = \sigma e(k+1). \tag{5}$$

The nominal system of system (2) is as follows:

$$x(k+1) = Ax(k) + Bu(k) + A_d x(k-\tau(k)). \tag{6}$$

The predicted output of the prediction model at the moment (k+P) can be obtained according to the nominal system (6).

$$\begin{aligned} s(k+p) =& \sigma e(k+P) = \sigma[y(k+P) - y_r(k+p)] \\ =& \sigma Cx(k+P) - \sigma y_r(k+P) \\ =& \sigma C[A^P x(k) + \sum_{i=1}^{P} A^{i-1} x(k+P-i \\ & - \tau(k+P-i)) + \sum_{i=1}^{M-1} A^{P-i} Bu(k+i-1) \\ & + \sum_{i=1}^{P-M} A^i Bu(k+M-1)] - \sigma y_r(k+P), \end{aligned} \tag{7}$$

where P is the prediction horizon, M is the control horizon and $M \le P$, $u(k+j) = u(k+M-1)$ $(j = M-1, \ldots, P)$. The vector form of (7) is as (8).

$$S_{PM}(k) = Gx(k) + HU(k) + FX_d(k) - \sigma Y_r(k), \tag{8}$$

where

$$\begin{aligned} & S_{PM} = [s(k+1), s(k+2), \cdots, s(k+P)]^T; \\ & U(k) = [u(k), u(k+1), \cdots, u(k+M-1)]^T; \\ & G = [(\sigma CA)^T, (\sigma CA^2)^T, \cdots, (\sigma CA^P)^T]^T; \\ & X_d(k) = [x(k-\tau(k)), x(k+1-\tau(k+1)), \\ & \qquad \cdots, x(k+P-1-\tau(k+P-1))]^T; \\ & Y_r(k) = [y_r(k+1), y_r(k+2), \cdots, y_r(k+P)]^T; \end{aligned}$$

$$F = \begin{bmatrix} \sigma CA_d & 0 & \cdots & \cdots & 0 \\ \sigma CAA_d & \sigma CA_d & & & 0 \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ \sigma CA^{P-1}A_d & \sigma CA^{P-2}A_d & \cdots & \cdots & \sigma CA_d \end{bmatrix};$$

$$H = \begin{bmatrix} \sigma CB & 0 & \cdots & \cdots & 0 \\ \sigma CAB & \sigma CB & 0 & \cdots & 0 \\ \sigma CA^{M-1}B & \vdots & \cdots & \cdots & \vdots \\ \vdots & \sigma CA^{M-2}B & \cdots & \sigma CAB & \sigma CB \\ \sigma CA^M B & \sigma CA^{M-1}B & \cdots & \sigma CA^2 B & \sigma CAB + \sigma CB \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \sigma CA^{P-1}B & \sigma CA^{P-2}B & \cdots & \sigma CA^{P-M+1}B & \sum_{i=0}^{P-M} \sigma CA^i B \end{bmatrix}.$$

3.1.2 Reference trajectory design

The reference trajectory of this paper is designed as follows:

$$\begin{cases} s_{ref} = (1 - \dfrac{s_0}{s_0 + |s(k)|}) s_{ref}(k) - \zeta(k) + \zeta_1, \\ s_{ref}(k) = s(k), \end{cases} \tag{9}$$

where the reference trajectory $s_(ref)(k)$ of present moment is equal to $s(k)$.

$$\zeta(k) = \sigma d(k). \tag{10}$$

Equality (10) represents the influence of faults, uncertainties and time delay of the system on sliding mode output value. Then take

$$\zeta_1 = \frac{\zeta_U + \zeta_L}{2} = \frac{\sigma d_U + \sigma d_L}{2}; \tag{11}$$

$$s_0 > \zeta_2 = \frac{\zeta_U - \zeta_L}{2} = \frac{\sigma d_U - \sigma d_L}{2}. \quad (12)$$

s_0 is the design constant, which can coordinate the control signal amplitude with the velocity that $s(k)$ converges to $s(k)=0$. Due to the interference of the uncertainty and the fault of the system, the interference suppression method is embedded in this reference trajectory, which uses ζ_1 compensates for $\zeta(k)$ to offset maximumly its impact on the system performance. When $|s(k)|$ is very small namely that $s(k)$ gradually enters the quasi sliding mode, $s_{ref}(k)$ approaches to $\lim_{k\to\infty} -\zeta(k)+\zeta_1 \to 0$ so the sliding mode chattering can be effectively suppressed.

Although $d(k)$ is bounded according to assumption 2, which may influence solving $s_{ref}(k+1)$. So one-step delay estimation method is used to obtain $\hat{d}(k)$ approximately as follow.

$$\begin{aligned}\hat{d}(k) =& d(k-1)\\ =& x(k)-Ax(k-1)-A_d x(k-1-\tau(k-1))\\ & -Bu(k-1)-Bu(k-1).\end{aligned}$$

The vector form of (9) is as follows:

$$S_{ref}(k) = [s_{ref}(k+1), s_{ref}(k+2), \cdots, s_{ref}(k+P)]^T, \quad (13)$$

where

$$\begin{aligned}s_{ref}(k+j) =& (1-\frac{s_0}{s_0+|s(k+j-1|)})s(k+j-1)\\ & -\zeta(k+j-1)+\zeta_1,\\ & j=1,2,\ldots,P.\end{aligned} \quad (14)$$

3.1.3 Feedback correction design

The predictive output of switching function of moment $(k-P)$ is defined as $s(k|k-P)$.

$$\begin{aligned}s(k|k-P) =& \sigma C[A^P x(k-P)\\ & +\sum_{i=1}^{P} A^{i-1}A_d x(k-i-\tau(k-i))\\ & +\sum_{i=1}^{M-1} A^(P-i)Bu(k-P+i-1)\\ & +\sum_{i=1}^{P-M} A^i Bu(k-P+M-1)] - \sigma y_r(k).\end{aligned} \quad (15)$$

Then the prediction error of moment k is as follow, where $s(k)$ is the actual output of switching function at moment k.

$$e_s(k) = s(k) - s(k|k-P). \quad (16)$$

The P step predictive output of switching function after adding the correction is as (17).

$$\tilde{s}(k+P) = s(k+P) + h_P e_s(k), \quad (17)$$

where h_P is the correction coefficient,and the vector form of (17) is as follows:

$$\tilde{S}_{PM}(k) = S_{PM}(k) + H_P E_s(k), \quad (18)$$

where

$$\tilde{S}_{PM}(k) = [\tilde{s}(k+1), \tilde{s}(k+2), \cdots, \tilde{s}(k+P)]^T;$$

$$H_P = \begin{bmatrix} h_1 & & & \\ & h_2 & & \\ & & \ddots & \\ & & & h_P \end{bmatrix};$$

$$\begin{aligned}E_s(k) = & [s(k)-s(k|k-1), s(k)-s(k|k-2),\\ & \cdots, s(k)-s(k|k-P)]^T.\end{aligned}$$

Generally, take $h_1 = 1$, $1 > h_2 > h_3 > \cdots > h_p > 0$. With the increase of the number of prediction steps, the effect of feedback correction is gradually weakened.

3.1.4 Receding-horizon optimization design

The receding-horizon optimization does not search the global optimal solutions, but the optimal control in the finite time horizon. The optimization performance index of moment k is (19).

$$\begin{aligned}j(k) =& \sum_{i=1}^{P} \beta_i [s_{ref}(k+i) - \tilde{s}(k+1)]^2\\ & +\sum_{l=1}^{M} \gamma_l [u(k+l-1)]^2,\end{aligned} \quad (19)$$

where β_i, γ_l are non-negative weights. β_i is the proportion of the sampling error in the performance index and γ_l is limit to control variables. The vector form of (19) is as follow:

$$\begin{aligned}J(k) =& [S_{ref}(k) - \tilde{S}_{PM}(k)]^T Q [S_{ref}(k) - \tilde{S}_{PM}(k)]\\ & +[U(k)]^T R [U(k)],\end{aligned} \quad (20)$$

where

$$Q = \begin{bmatrix} \beta_1 & & & \\ & \beta_2 & & \\ & & \ddots & \\ & & & \beta_P \end{bmatrix}, \quad R = \begin{bmatrix} \gamma_1 & & & \\ & \gamma_2 & & \\ & & \ddots & \\ & & & \gamma_M \end{bmatrix}.$$

The improved multi agent PSO is used to determine $U(k)$. Let the optimization performance index $J(k)$ be the fitness function Ψ.

Assume that particle swarm size is L. The position and the velocity of the i^{th} swarm is respectively $u_i = (u_{i1}, u_{i2}, \cdots, u_{iM})$ and $v_i = (v_{i1}, v_{i2}, \cdots, v_{iM})$. The best position of the i^{th} swarm is $p_i = (p_{i1}, p_{i2}, \cdots, p_{iM})$. The best fitness value of the i^{th} swarm is Ψ_{ibest} and the current fitness value of the i^{th} is Ψ_i. The neighoring particles of the i^{th} swarm are all particles whose position are in

$\{(n_{i1}, n_{i2}, \cdots, n_{iM})| \quad |n_{ij} - u_{ij}| \le \delta, j = 1, 2, \cdots, M\}$. δ is the particle environment range.

The improved iterative formulas are as follows:

$$\begin{cases} v_i^{t+1} = wv_i^t + c_1 r_1 (p_i - u_i^t) + c_2 r_2 (g - u_i^t), \\ u_i^{t+1} = u_i^t + v_i^{t+1}, \\ w = w_{min} + [(t_{max} - t)(w_{max} - w_{min})]/t_{max}, \end{cases} \tag{21}$$

where c_1, c_2 are learning factors and r_1, r_2 are random numbers between 0 and 1. $g = (g_1, g_2, \cdots, g_p)$ is the global optimal position which is the optimal control quantity required. w is the weight the coefficient and t_{max} is the maximum number of iteration. When w takes a smaller value, the convergence velocity of particle swarm would be accelerated. When w takes a larger value, local optimum can be avoided effectively. In this paper, the dynamic weight is selected, which can balance accuracy and convergence velocity. Concrete implementation steps are in part 3.3.

3.2. Stability analysis

For practical systems (1), let moment k be current moment and the prediction output at moment $(k+P)$ is (22).

$$\begin{aligned} s(k+P) =& \sigma C[A^P x(k) \\ &+ \sum_{i=1}^{P} A^{i-1} A_d x(k+P-i-\tau(k+P-i)) \\ &+ \sum_{i=1}^{M-1} A^{P-i} Bu(k+i-1) \\ &+ \sum_{i=1}^{P-M} A^i Bu(k+M-1) \\ &+ \sum_{i=1}^{P} A^{i-1} d(k+i-1) - \sigma y_r(k+P) \end{aligned} \tag{22}$$

The vector form of (22) is as follows:

$$\begin{aligned} S_{PM}(k) =& Gx(k) + HU(k) + FX_d(k) \\ &+ KD(k) - \sigma Y_r(k), \end{aligned} \tag{23}$$

where

$$K = \begin{bmatrix} \sigma C & 0 & \cdots & 0 \\ \sigma CA & \sigma C & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma CA^{P-1} & \sigma CA^{P-2} & \cdots & \sigma C \end{bmatrix};$$

$$D(k) = [d(k), d(k+1), \cdots, d(k+P-1)]^T.$$

The necessary condition for the extreme value of $J(k)$ is $\frac{\partial J(k)}{\partial U(k)} = 0$, so the control law calculated through the particle swarm optimization must meet $\frac{\partial J(k)}{\partial U(k)} = 0$. Therefore, $U(k)$ must meet (24).

$$\begin{aligned} U(k) =& (R + H^T QH)^{-1} H^T Q[S_{ref}(k) - Gx(k) \\ &- FX_d(k) + \sigma Y_r(k) + H_p E_s(k)]. \end{aligned} \tag{24}$$

Equation (25) can be obtained by substituting (24) into (23).

$$\begin{aligned} S_{PM}(k) =& Gx(k) + H[(R + H^T QH)^{-1} H^T Q \\ &\times [S_{ref}(k) - Gx(k) - FX_d(k) + \sigma Y_r(k) \\ &+ H_p E_s(k)]] + FX_d(k) + KD(k) - \sigma Y_r(k). \end{aligned} \tag{25}$$

The weight coefficient matrix R is the constraint for $U(k)$, and generally let $R = 0$ when analyzing robust stability. Then the following statement can be obtained.

$$S_{PM}(k) = S_{ref}(k) + H_p E_s(k) + KD(k). \tag{26}$$

In the Receding-horizon optimization process, only the current control input signal is implemented to the controlled object, so the actual sliding mode switching function can be written as follows:

$$\begin{aligned} s(k+1) &= [1, 0, \cdots, 0] S_{PM}(k) \\ &= s_{ref}(k+1) + h_1[s(k) - s(k|k-1)] \\ &\quad + \sigma Cd(k) \\ &= s_{ref}(k+1) + \sigma C[d(k) - h_1 d(k-1)]. \end{aligned} \tag{27}$$

Generally, let $h_1 = 1$, and then (27) can be written as (28).

$$s(k+1) = s_{ref}(k+1) + \sigma C[d(k) - d(k-1)]. \tag{28}$$

Definition 1: In discrete time sliding mode control, a quasi sliding mode is considered in the vicinity of the sliding surface, such that $|s(k) < \varepsilon|$ where $s(k)$ is the sliding function and ε is a positive constant called the quasi sliding mode band width [19, 21].

A brief illustration is proposed to explain that the reference trajectory $s_{ref}(k+1)$ must be able to reach and maintain the quasi sliding mode in finite time as follows:

1) $|s_{ref}(k_0)| > \frac{s_0 \zeta_2}{s_0 - \zeta_2} = \varepsilon$.

1.1) When $s_{ref}(k_0) = \frac{s_0 \zeta_2}{s_0 - \zeta_2} + \varphi > \varepsilon, \quad \varphi > 0$.

$$\begin{aligned} &\Delta s_{ref}(k_0) \\ &= s_{ref}(k_0+1) - s_{ref}(k_0) \\ &= -\frac{[s_0^2 + \varphi(s_0 - \zeta_2)][\zeta_2 + \zeta(k) - \zeta_1]}{s_0^2 + \varphi[s_0 - \zeta_2]} \\ &= -\frac{s_0^2[\zeta_2 + \zeta(k) - \zeta_1]}{s_0^2 + \varphi[s_0 - \zeta_2]} \\ &\quad - \frac{\varphi(s_0 - \zeta_2)[\zeta_2 + \zeta(k) - \zeta_1]}{s_0^2 + \varphi[s_0 - \zeta_2]} \\ &\le -\frac{\varphi(s_0 - \zeta_2)[\zeta_2 + \zeta(k) - \zeta_1]}{s_0^2 + \varphi[s_0 - \zeta_2]} < 0. \end{aligned} \tag{29}$$

When $s_{ref}(k)$ is positive, $s_{ref}(k)$ is the decreasing function on k.

Remark 1: When $s_{ref}(k_0)$ initial state is not on quasi sliding mode and $s_{ref}(k_0)$ is positive, $s_{ref}(k)$ will gradually decrease until entering the quasi sliding mode.

1.2) When $s_{red}(k_0) = -(\frac{s_0\zeta_2}{s_0-\zeta_2}+\varphi) < -\varepsilon, \quad \varphi > 0,$

$$\begin{aligned}
&\Delta s_{ref}(k_0)\\
&= s_{ref}(k_0+1) - s_{ref}(k_0)\\
&= \frac{[s_0^2+\varphi(s_0-\zeta_2)][\zeta_2+\zeta(k)-\zeta_1]}{s_0^2+\varphi[s_0-\zeta_2]}\\
&= \frac{s_0^2[\zeta_2+\zeta(k)-\zeta_1]}{s_0^2+\varphi[s_0-\zeta_2]}\\
&\quad + \frac{\varphi(s_0-\zeta_2)[\zeta_2+\zeta(k)-\zeta_1]}{s_0^2+\varphi[s_0-\zeta_2]}\\
&\geq \frac{\varphi(s_0-\zeta_2)[\zeta_2+\zeta(k)-\zeta-1]}{s_0^2+\varphi[s_0-\zeta_2]} > 0.
\end{aligned} \tag{30}$$

When $s_{ref}(k)$ is negative, $s_{ref}(k)$ is the increasing function on k.

Remark 2: When $s_{ref}(k_0)$ initial state is not on quasi sliding mode and $s_{ref}(k_0)$ is negative,$s_{ref}(k)$ will gradually increase until entering the quasi sliding mode.

2) $|s_{ref}(k_0)| \leq \frac{s_0\zeta_2}{s_0-\zeta_2} = \varepsilon.$

$$\begin{aligned}
|s_{ref}(k_0+1)| &\leq \frac{s_0^2\zeta_2^2/(s_0-\zeta_2)^2}{(s_0\zeta_2/s_0-\zeta_2)+s_0}+\zeta_2\\
&= \frac{\zeta_2^2}{s_0-\zeta_2}+\zeta_2 = \frac{s_0\zeta_2}{s_0-\zeta_2}.
\end{aligned} \tag{31}$$

Remark 3: For any $s_{ref}(k), k > k_0$, $|s_{ref}(k)| \leq \frac{s_0\zeta_2}{s_0-\zeta_2} = \varepsilon$ is always true. As long as the quasi sliding mode is reached, it must be kept in this state.

The following statement can be obtained from Assumption 1.

$$\sigma C|d(k)-d(k-1)| \leq \sigma C d_0. \tag{32}$$

Then there must be a certain moment k_0, $|s(k+1)| = |s_{ref}(k+1)+\sigma C[d(k)-d(k-1)]| \leq \varepsilon + \sigma C w_0$ can be obtained when $k > k_0$. Therefore, the controller designed in this paper is robust stable.

3.3. Algorithm implementation steps

For the system described by the formula (1), the implementation steps of the sliding mode fault tolerant predictive control based on particle swarm optimization are as follows.

Step 1: Initialize the system state. Determine each parameter of the object system. Set the prediction horizon, the control horizon and each weight coefficients.

Step 2: Initialize the particle swarm. Set the size, environment range, maximum number of iteration, range of weight, learning factor of the particle swarm.

Step 3: Update positions of particles according to the information of neighboring particles. Assume that n is the particle of the neighboring particle of i^{th} particle with optimal fitness value. If the fitness value of i^{th} particle is better than that of n, maintain the position of i^{th} particle. Otherwise, update the position of i^{th} particle according to equation (33), where $\xi \in [-1,1]$ is a random number.

$$u_i' = u_n + \xi(u_i - u_n). \tag{33}$$

Step 4: Iterate particles. The position and velocity of the particles are updated iteratively, and the optimal position of the population is obtained according to formula (21).

Step 5: End optimization. The conditions of optimization finish: i)Reach the maximum number of iteration; ii)Within the allowable error range.

Step 6: Implement the current control law and let $k+1 \rightarrow k$ return Step 1.

4. EXPERIMENT RESULT

In this section, the Qball-X4 quad-rotor helicopter developed by Quanser in Canada [22] is used as the research object as Fig. 1. There are six dimensional variables (X,Y,Z,ψ,θ,ϕ) in the system, where X,Y,Z are position variables and ψ is the yaw angle, θ is the pitch angle, ϕ is the roll angle. The X axis direction is chosen as the research object.

The motion of the Qball-X4 along the X axis is caused by the total thrust and by changing roll/pitch angles. Assuming that the yaw angle is zero the dynamics of motion in X axis can be written as

$$M_g\ddot{X} = 4F\sin(\dot{\theta}),$$

where M_g is the total msaa of the device, X is the position of X axis direction, F is the generated by each propeller, and it is modeled using the following first-order system.

$$F = K_g\frac{\omega}{s+\omega}u,$$

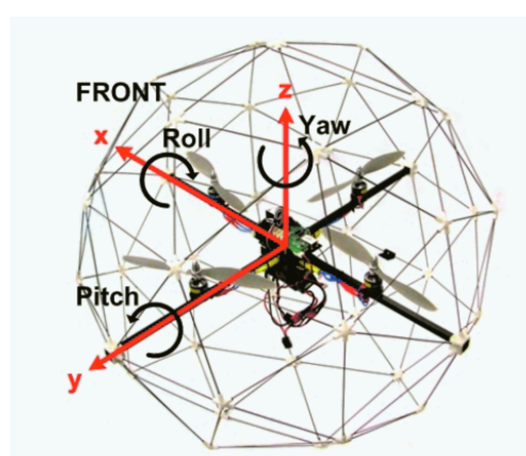

Fig. 1. The main body of Qball-X4.

where u is PWM input to the actuator, ω is the actuator bandwidth and K_g is a positive gain. A state variable, v will be used to represent the actuator dynamics, which is defined as follows:

$$v = \frac{\omega}{s+\omega}u.$$

The pitch angle is coupled with the X axis position control model, so the whole control process can be divided into two stages. First period is pitch angle control process to control pitch angle to preset values. And then to start position control period, in which the pitch angle would be set close to zero when the position reaches the setting position. In the case of small pitch angle, X axis direction model by linearization without external disturbance, parameter perturbation and time varying delay is obtained as follow:

$$\begin{bmatrix} \dot{X} \\ \ddot{X} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \frac{4K_g}{M_g}\theta \\ 0 & 0 & -\omega \end{bmatrix} \begin{bmatrix} X \\ \dot{X} \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \omega \end{bmatrix} u. \tag{34}$$

The each matrix of system (1) can be described as below when the disturbance, perturbation, time varying delay and fault are introduced.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 12 \\ 0 & 0 & -10 \end{bmatrix}, \ A_d = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 15 \end{bmatrix}^T, \ E = \begin{bmatrix} 0.1 & 0.1 & 0.2 \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \Delta A = 0.1A, \quad \Delta B = 0.1B,$$

$$\Delta A_d = 0.1A_d, \quad x(0) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T,$$

$$f(k) = 1.5 + \begin{bmatrix} 0.3\sin(6k) & 0 & 0.2\sin(2k) \end{bmatrix} x(k),$$

elements of $v(k)$ are Gauss white noise with zero mean value, $\sigma = [1]$, The particle swarm learning factor $c_1 = 2$, $c_2 = 2$, range of weight $w_{min} = 0.2$, $w_{max} = 0.9$, $L = 20$, maximum number of iteration $t_{max} = 50$, the particle environment range $\delta = 6$. The prediction horizon P indicates that outputs at present moment to the expected values of P steps to the future, which should cover the main part of the dynamic influence of the controlled object. Experimentation and practice show that the response of system is slow and the stability of the system is enhanced when P is increased. The situation is just the opposite when P is decreased. So this paper chooses the prediction horizon $P = 4$, which takes account of both the fast and the stability of the system. The control horizon M represents the number of changes of control variables to be determined in the future. The effect of M on the system is opposite to that of P. The general selection of M is 1∼2 for a system with not very complex dynamic characteristics, so this paper chooses $M = 2$. $\tau(k)$ is the integer belonging to $[1,3]$, and simulation time domain $k = 1000$. Where parameters of Qball-X4 are $K_g = 120$ N, $\omega = 15$ rad/s, $M_g = 1.4$ kg.

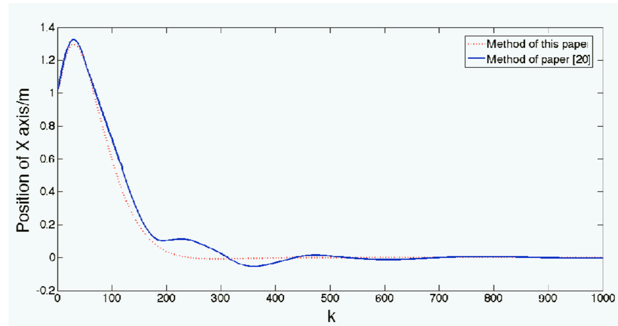

Fig. 2. The position trajectories of X-axis.

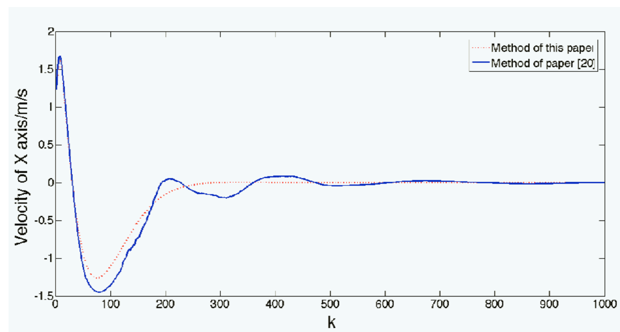

Fig. 3. The velocity trajectories of X-axis.

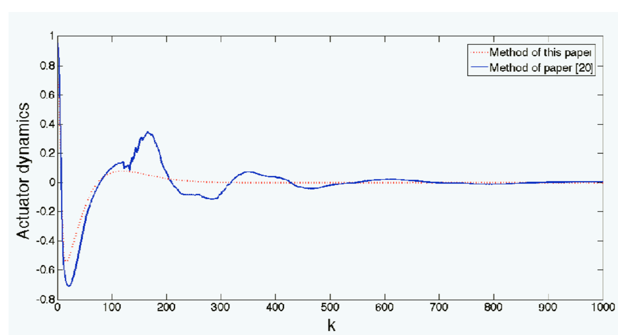

Fig. 4. The actuator dynamics trajectories of X-axis.

From Fig. 2 to Fig. 4, it is not difficult to see that the system with the disturbance, perturbation, time varying delay and fault can be quickly stabilized by the control method proposed in this paper. Compared to the sliding mode predictive control proposed in [20], the trajectories of quad-rotor helicopter used by the control method proposed in this paper are smoother. The helicopter is still stable and safe under actuator fault. Figs. 5 and 6 show that the control law is fast convergent. Compared to the other two control methods, the chattering amplitude of the control law designed in this paper is reduced by almost 50%. It is known that the fault tolerant control method designed in this paper is effective for the system with the disturbance, perturbation, time varying delay and actuator fault from the above experimental results.

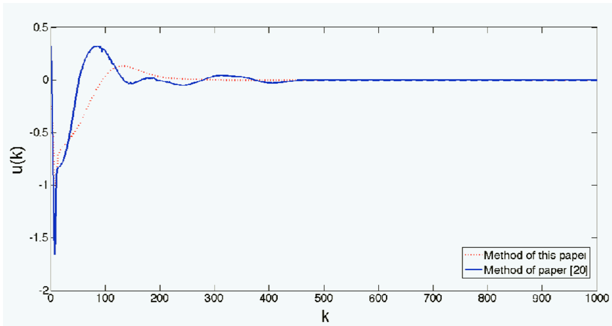

Fig. 5. The trajectories of control law (I).

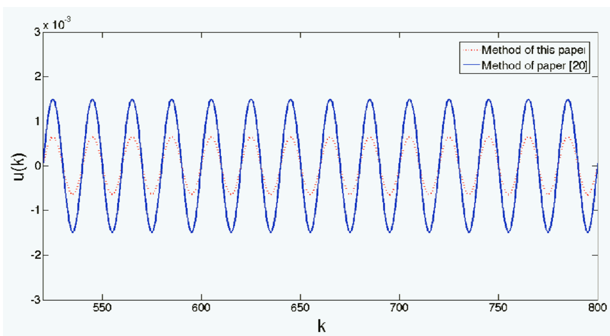

Fig. 6. The trajectories of control law (II).

5. CONCLUSION

Combined with the advantages of robustness of SMC and on-line real time optimization of MPC a sliding mode fault tolerant predictive control based on multi agent PSO is presented for uncertain discrete-time systems with time varying delay under actuator faults. Simulation results show that the proposed control scheme has fast convergence and robustness. Chattering phenomenon is inhibition effectively and dynamic quality of the system is good. Further studies will focus on eliminating upper bound of uncertainties and time delay.

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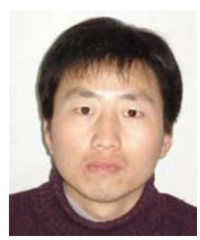

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